Commutative, Distributive, and Associative properties

Commutative property:

For multiplication:  \( x \cdot y = y \cdot x \)

**Example 1:**

\[ 7 \cdot 9 = 9 \cdot 7 \]

For addition:  \( x + y = y + x \)

**Example 2:**

\[ 8 + 2 = 2 + 8 \]

Notice that numbers **don’t commute** under the operation of subtraction:

\[ 4 - 3 \neq 3 - 4 \]

Distributive property:  The product of a number and a sum is equal to the sum of the individual products of addends and the number.

**Example 3:**

\[ 3(5 + 11) = 3 \cdot 5 + 3 \cdot 11 \]

**Example 4:**

\[ a(b + c) = ab + ac \]
**Associative property:** The addition or multiplication of a several numbers is the same regardless of how the numbers are grouped. The associative property will always involve 3 or more numbers. The parenthesis groups the terms that are considered one unit.

**Associative property of addition:**
Example 5:

\[ 5 + (7 + 3) = (5 + 7) + 3 \]
\[ (x + y) + z = x + (y + z) \]

**Associative property of multiplication:**
Example 6:

\[ (4 \cdot 7) \cdot 3 = 4 \cdot (7 \cdot 3) \]
\[ a \cdot (b \cdot c) = (a \cdot b) \cdot c \]

**Example 7:** Name the properties illustrated by these equations:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 8x + 2y = 2y + 8x )</td>
<td>Commutative property of addition</td>
</tr>
<tr>
<td>( 7 + (5 + 9) = (7 + 5) + 9 )</td>
<td>Associative property of addition</td>
</tr>
<tr>
<td>( 4 + 19 = 19 + 4 )</td>
<td>Commutative property of addition</td>
</tr>
<tr>
<td>( 5(8 + 3) = 5 \cdot 8 + 5 \cdot 3 )</td>
<td>Distributive property</td>
</tr>
<tr>
<td>( (x + y) + z = x + (y + z) )</td>
<td>Associative property of addition</td>
</tr>
<tr>
<td>( (a + b)x = ax + bx )</td>
<td>Distributive property</td>
</tr>
<tr>
<td>( 3x(2y) = (2y)3x )</td>
<td>Commutative property of multiplication</td>
</tr>
<tr>
<td>( 5 \cdot (9 \cdot 3) = (5 \cdot 9) \cdot 3 )</td>
<td>Associative property of multiplication</td>
</tr>
</tbody>
</table>
Assignment: Name the properties illustrated by these equations:

1. \(11 \cdot 4 = 4 \cdot 11\)

2. \(127(x + y + z) = 127x + 127y + 127z\)

3. \(1 + (2 + 3 + 4) = (1 + 2 + 3) + 4\)

4. \(3 \cdot 5 + 8 \cdot 5 + 4 \cdot 5 = (3 + 8 + 4)5\)

5. \(f + g = g + f\)

6. \(p \cdot q = q \cdot p\)
7. \( m \cdot (n \cdot p) \cdot q = m \cdot n \cdot (p \cdot q) \)

8. \( a \cdot b \cdot c = b \cdot a \cdot c \)

9. \( 115 \cdot (59 \cdot 19) = (115 \cdot 59) \cdot 19 \)

10. \( (47 - 11)x = 47x - 11x \)

*11. \( (76 - x) \cdot (a + b) = (a + b) \cdot (76 - x) \)

*12. \( (76 - x) + (a + b) = (a + b) + (76 - x) \)